Chapter 24
The Method of Earth Rotation Parameter Determination Using GNSS Observations and Precision Analysis

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Abstract Based on the principle of GNSS observation, the linear observation equations of earth rotation parameters (ERP) estimation using GNSS data are deduced. The general solution and precision evaluation function are obtained. According to the precision evaluation function, the major factors of effect ERP solution precision are analyzed. One is the geometric distribution of the satellites relative to the station; another is the geometric distribution of the stations relative to the geo-center. It is proved that ERP will not be solved if using one GNSS station. The solution precision of ERP will be the best when the vectors of two stations to geo-center are orthogonal, if two stations are used. For testing ERP estimation precision using GNSS data, two weeks GPS data from 295 and 21 IGS stations are processed, respectively. The results show that the precise ERP can be obtained whether 295 or 21 stations data are used, when the stations rational distribution. The estimation precisions of pole motion and length of day parameters are better than 0.04 mas and 0.03 ms respectively, where the results of IERS are taken as the reference values.

Keywords Global navigation satellite system · Earth rotation parameter · Linear observation equation · Precision factors · Optimal configuration

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24.1 Introduction

The earth rotation parameters (ERP) includes that the pole motion (PM) and length of day (LOD), which with the precession and nutation constitute together the earth orientation parameters (EOP). It is the most import theory basis of setting up high precision celestial reference system and terrestrial reference system. And it is necessary for the artificial satellite orbit determination, the autonomous navigation of space craft and high accurate time service. Additionally, ERP includes the potential information of the earth interior material motion and effect of the other celestial bodies on the earth motion. Therefore, ERP research attracts the great attention of geodesy, geodynamics, geophysics, astronomy.

The classical optical instruments were used at early stage of ERP observation, which have a precision of ± 1 m [1]. In the 1970s, the ERP observation precision had been improved by two orders of magnitude, with the rising of modern space measurement technology, such as VLBI, SLR, LLR and DORIS [2]. However, the temporal resolution of ERP is poor, because of the huge equipment and the low sample rate of VLBI and SLR [3]. It is not suitable for applications of high temporal and spatial resolution. Therefore, it becomes a key problem of improving the temporal resolution of ERP.

The GNSS technology has a wide station distribution, low equipment cost, high observation precision and high data sample rate. Therefore, it is very suitable for obtaining high precision and high temporal resolution of ERP [4–6]. With the building up of Compass satellite navigation system, it is very important of developing ERP monitoring system based on Compass observation for setting up Chinese independent space-time reference system. In this paper, the linear observation equations of ERP estimation using GNSS observation are deduced, as well as the general solution and precision evaluation function have been obtained. The major factors of effect ERP estimation precision are analyzed. Additionally the actual precision of ERP solution using GNSS data has been tested by the real GNSS data from 295 IGS stations.

24.2 Method of ERP Determination Using GNSS Observations

Based on the principle of GNSS observation, the GNSS observation equation can be written as [7]:

$$\rho + \triangle\rho = \bar{\rho} \tag{24.1}$$

where ρ is the observation distance between satellite and station; $\triangle\rho$ is the equivalent distance of observation error; $\bar{\rho}$ is the theory distance. The concrete expressions of three items as follows:

$$\rho = \lambda(\varphi + N) \tag{24.2}$$

$$\triangle\rho = dt_s + dt_r + dtrop + diono + dmulti + drel + d\varepsilon \tag{24.3}$$

$$\bar{\rho} = |RX_s - RX_r| \tag{24.4}$$

where $\lambda,\ \varphi,\ N$ are the wavelength, observation, integer ambiguity of carrier phase observation, respectively; $dt_s,\ dt_r,\ dtrop,\ diono,\ dmulti,\ drel,\ d\varepsilon$ are the equivalent distances of satellite clock error, receiver clock error, troposphere error, ionosphere error, multipath error, relativistic error and observation noise, respectively; $X_s,\ X_r$ are the coordinates of satellite and receiver in the terrestrial reference system; R is the coordinate transformation matrix from terrestrial reference system to celestial reference system, which can be expressed as:

$$R = PNSU \tag{24.5}$$

where *P, N, S, U* are the rotation matrix of precession, nutation, sidereal time and polar motion. The detailed calculation formula of *PNSU* can see the references [8]. The length of day and polar motion parameters $\left(\theta_t,\ \theta_x,\ \theta_y\right)$ are included in the rotation matrix S and U. Therefore, the observation equation needs to be linearized if estimating ERP using GNSS data. Using the Taylor series expansion to one order, Eq. (24.1) can be written as:

$$\rho + \triangle\rho = \bar{\rho}_0 + \frac{\delta\bar{\rho}}{\delta\theta_0}\delta\theta \tag{24.6}$$

where $\bar{\rho}_0$ is the approximate value using the initial ERP; $\delta\theta$ includes $\delta\theta_t, \delta\theta_x, \delta\theta_y$; the expressions of $\frac{\delta\bar{\rho}}{\delta\theta_0}$ as follows:

$$\frac{\delta\bar{\rho}}{\delta\theta_{x_0}} = -\frac{(R_0X_s - R_0X_r)^T}{\bar{\rho}_0}PNS_0\frac{\delta U}{\delta\theta_{x_0}}X_r \tag{24.7}$$

$$\frac{\delta\bar{\rho}}{\delta\theta_{y_0}} = -\frac{(R_0X_s - R_0X_r)^T}{\bar{\rho}_0}PNS_0\frac{\delta U}{\delta\theta_{y_0}}X_r \tag{24.8}$$

$$\frac{\delta\bar{\rho}}{\delta\theta_{t_0}} = -\frac{(R_0X_s - R_0X_r)^T}{\bar{\rho}_0}PN\frac{\delta S}{\delta\theta_{t_0}}U_0X_r \tag{24.9}$$

where $R_0,\ S_0,\ U_0$ are the approximate values of corresponding matrix based on the initial ERP. Ignoring the effect of micro items, $\frac{\delta U}{\delta\theta_{x_0}}, \frac{\delta U}{\delta\theta_{y_0}}, \frac{\delta U}{\delta\theta_{t_0}}$ can be written as:

$$\frac{\delta U}{\delta\theta_{x_0}} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \tag{24.10}$$

$$\frac{\delta U}{\delta\theta_{y_0}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \tag{24.11}$$

$$\frac{\delta U}{\delta\theta_{t_0}} = \begin{bmatrix} -\sin(GAST) & -\cos(GAST) & 0 \\ \cos(GAST) & -\sin(GAST) & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \gamma \times (t - t_0) \tag{24.12}$$

where $GAST$ is Greenwich apparent sidereal time; the calculation formula of $GAST$ see the references [9]. $\gamma = 1.0027379093$; t is the Julian date of observation time, and t_0 is the Julian date of reference time.

Based on the above formulas, the linear observation equations of ERP estimation using GNSS data can be obtained, when the initial values of ERP and observation time are given. If assuming there are n stations and m satellites are observed by each station, the observation equation set is written as:

$$A_{(n\times m)\times 3}X_{3\times 1} = L_{(n\times m)\times 1},\ P_{(n\times m)(n\times m)} \tag{24.13}$$

where A is the coefficient matrix; L is the constant matrix; X is the unknown parameter matrix; P is the weighting matrix; the expressions of each matrix as follows:

$$A_{(n\times m)\times 3} = \begin{bmatrix} \frac{\delta\overline{\rho_1^1}}{\delta\theta_{x_0}} & \frac{\delta\overline{\rho_1^1}}{\delta\theta_{y_0}} & \frac{\delta\overline{\rho_1^1}}{\delta\theta_{t_0}} \\ \cdots & \cdots & \cdots \\ \frac{\delta\overline{\rho_i^j}}{\delta\theta_{x_0}} & \frac{\delta\overline{\rho_i^j}}{\delta\theta_{y_0}} & \frac{\delta\overline{\rho_i^j}}{\delta\theta_{t_0}} \\ \cdots & \cdots & \cdots \\ \frac{\delta\overline{\rho_n^m}}{\delta\theta_{x_0}} & \frac{\delta\overline{\rho_n^m}}{\delta\theta_{y_0}} & \frac{\delta\overline{\rho_n^m}}{\delta\theta_{t_0}} \end{bmatrix} \tag{24.14}$$

$$L_{(n\times m)\times 1} = \begin{bmatrix} \rho_1^1 + \triangle\rho_1^1 - \overline{\rho_1^1} \\ \cdots \\ \rho_i^j + \triangle\rho_i^j - \overline{\rho_i^j} \\ \cdots \\ \rho_n^m + \triangle\rho_n^m - \overline{\rho_n^m} \end{bmatrix} \tag{24.15}$$

$$X_{3\times 1} = \begin{bmatrix} \delta\theta_x \\ \delta\theta_y \\ \delta\theta_t \end{bmatrix} \tag{24.16}$$

If assuming k epochs are observed, the k normal equations are added together and the iterative least square adjustment is used to estimate the ERP. The solution result and variance-covariance matrix as follows:

$$X = \Big(\sum_{i=1}^{i=k} A_i^T P_i A_i\Big)^{-1} \sum_{i=1}^{i=k} A_i^T P_i L_i \tag{24.17}$$

$$D = \sigma_0^2 \Big(\sum_{i=1}^{i=k} A_i^T P_i A_i\Big)^{-1} \tag{24.18}$$

$$\sigma_0^2 = \frac{V^T V}{n - t} \tag{24.19}$$

where n is the number of observation equation; t is the number of unknown parameter; V is the residual matrix; σ_0^2 is the variance of unit weight.

24.3 Major Factors of Effect ERP Solution Precision

For the simplicity of discussion, the initial values of ERP and observation time are given, where $\theta_{x_0} = 0.11881''$, $\theta_{y_0} = 0.26326''$, $\theta_{t_0} = 0.001355s/d$, $t = 2455927.5$, $t = 2455926.5$, and the mica item (10^{-6}) is ignored. The Eqs. (24.7–24.9) can be written as:

$$\frac{\delta\bar{\rho}}{\delta\theta_{x_0}} = \frac{\begin{bmatrix} x_s - x_r & y_s - y_r & z_s - z_r \end{bmatrix}}{\bar{\rho}_0} \begin{bmatrix} -z_r \\ 0 \\ x_r \end{bmatrix} \tag{24.20}$$

$$\frac{\delta\bar{\rho}}{\delta\theta_{y_0}} = \frac{\begin{bmatrix} x_s - x_r & y_s - y_r & z_s - z_r \end{bmatrix}}{\bar{\rho}_0} \begin{bmatrix} 0 \\ z_r \\ -y_r \end{bmatrix} \tag{24.21}$$

$$\frac{\delta\bar{\rho}}{\delta\theta_{t_0}} = \frac{\begin{bmatrix} x_s - x_r & y_s - y_r & z_s - z_r \end{bmatrix}}{\bar{\rho}_0} \begin{bmatrix} y_r \\ -x_r \\ 0 \end{bmatrix} \tag{24.22}$$

If assuming m satellites are observed by one station, the coefficient matrix A can be written as:

$$A_{m\times 3} = \begin{bmatrix} \frac{x_s^1 - x_r}{\bar{\rho}_0} & \frac{y_s^1 - y_r}{\bar{\rho}_0} & \frac{z_s^1 - z_r}{\bar{\rho}_0} \\ \cdots & \cdots & \cdots \\ \frac{x_s^j - x_r}{\bar{\rho}_0} & \frac{y_s^j - y_r}{\bar{\rho}_0} & \frac{z_s^j - z_r}{\bar{\rho}_0} \\ \cdots & \cdots & \cdots \\ \frac{x_s^m - x_r}{\bar{\rho}_0} & \frac{y_s^m - y_r}{\bar{\rho}_0} & \frac{z_s^m - z_r}{\bar{\rho}_0} \end{bmatrix} \begin{bmatrix} -z_r & 0 & y_r \\ 0 & z_r & -x_r \\ x_r & -y_r & 0 \end{bmatrix} \tag{24.23}$$

The Eq. (24.23) can be simplified written as:

$$A_{m\times 3} = B_{m\times 3} C_{3\times 3} \tag{24.24}$$

Based on the Eq. (24.24), it can be known that the coefficient matrix A of ERP solution includes two parts: one is the direction cosine matrix of satellite and station; another is the coordinate matrix of stations. If assuming the weighting matrix P is the unit matrix, the precision matrix Q can be written as:

$$Q = (A^T A)^{-1} = (C^T B^T BC)^{-1} \tag{24.25}$$

From Eq. (24.25), it can be drawn that the major factors of effect ERP estimation precision are the geometric distribution of the satellites relative to the station and the geometric distribution of the stations relative to the geo-center.

If assuming $B^T B$ is the unit matrix, namely without considering of effect of satellite, the matrix $A^T A$ can be expressed as:

$$A^T A = C^T C = \begin{bmatrix} x_r^2 + z_r^2 & -x_r y_r & -z_r y_r \\ -x_r y_r & z_r^2 + y_r^2 & -z_r x_r \\ -z_r y_r & -z_r x_r & y_r^2 + x_r^2 \end{bmatrix} \tag{24.26}$$

According to the Eq. (24.26), the determinant of $A^T A$ can be calculated ($\det(A^T A) = 0$). The result shows the ERP can not be estimated if using one station. If there are two stations and m satellites are observed by each station, the matrix A can be written as:

$$A_{2m\times 3} = \begin{bmatrix} B_1 C_1 \\ B_2 C_2 \end{bmatrix} \tag{24.27}$$

where $B_1,\ C_1,\ B_2,\ C_2$ are the direction cosine matrix and coordinate matrix of two stations. And assuming (x_1, y_1, z_1) and (x_2, y_2, z_2) are the coordinates of two stations. The matrix $A^T A$ can be expressed as:

$$A^T A = C_1^T C_1 + C_2^T C_2 \tag{24.28}$$

$$A^T A = \begin{bmatrix} x_1^2 + z_1^2 + x_2^2 + z_2^2 & -(x_1 y_1 + x_2 y_2) & -(z_1 y_1 + z_2 y_2) \\ -(x_1 y_1 + x_2 y_2) & z_1^2 + y_1^2 + z_2^2 + y_2^2 & -(z_1 x_1 + z_2 x_2) \\ -(z_1 y_1 + z_2 y_2) & -(z_1 x_1 + z_2 x_2) & y_1^2 + x_1^2 + y_2^2 + x_2^2 \end{bmatrix} \tag{24.29}$$

If two stations are very close, namely $x_1 \approx x_2,\ y_1 \approx y_2,\ z_1 \approx z_2$, the determinant of matrix $A^T A$ is zero. The necessary and sufficient condition of ERP can be estimated by the Eq. (24.17) is $\det(A^T A) > 0$. And the solution precision is improving with the value of $\det(A^T A)$ is increasing. It can be proved that the value of determinant is maximal when the non diagonal elements are zero and the diagonal elements are equal [10]. Therefore, some condition equations can be obtained as follows:

$$x_1 y_1 = -x_2 y_2 \tag{24.30}$$

$$z_1 y_1 = -z_2 y_2 \tag{24.31}$$

$$z_1x_1 = -z_2x_2 \tag{24.32}$$

$$x_1^2 + z_1^2 + x_2^2 + z_2^2 = z_1^2 + y_1^2 + z_2^2 + y_2^2 \tag{24.33}$$

Based on above equations, if the coordinates of one station are given, the coordinates of another station will can be determined. For example, the coordinates of one station are $(r, 0, 0)$, the coordinates of another station are $(0, \pm r, 0)$. Therefore, it can be known that the solution precision of ERP will be the best when the vectors of two stations to geo-center are orthogonal.

If the number of stations is larger than two, the optimal condition of stations distribution for ERP estimation is more complicated than that. In this paper, it is not discussed.

24.4 Data Processing and Analysis

For testing ERP estimation precision using GNSS data, two weeks GPS data from 295 IGS stations are processed (11th–25th September 2007). For analysis of effect of station distribution on ERP solution, the data of 21 stations are tested, which are chosen from above stations and have a rational distribution.

The coordinates of stations, the orbit parameters of satellites, light pressure parameters are constrained strongly in the data processing. One set of ERP are estimated every 24 h and the results are compared with the value of IERS. Figure 24.1 shows the location of 21 tested stations. Figures 24.2, 24.3, 24.4 show the difference between the experimental results and the IERS results about pole motion and length of day, respectively. Table 24.1 is the precision statistics of ERP solution in two experiments.

Based on above experimental results, it can be known that there is light effect of satellite distribution on ERP estimation because the long time observation will

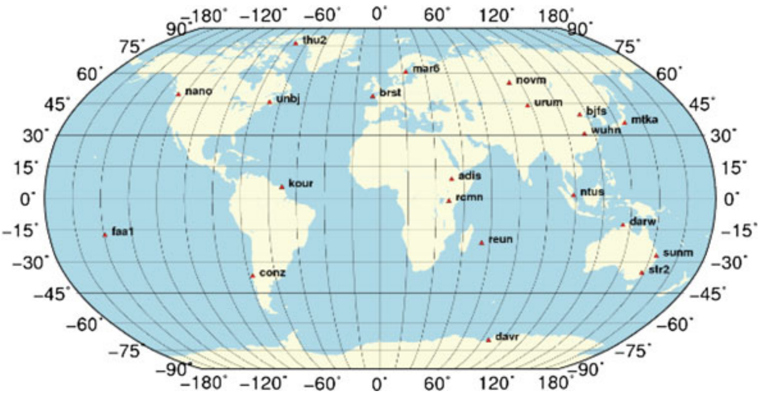

Fig. 24.1 The location of 21 tested stations

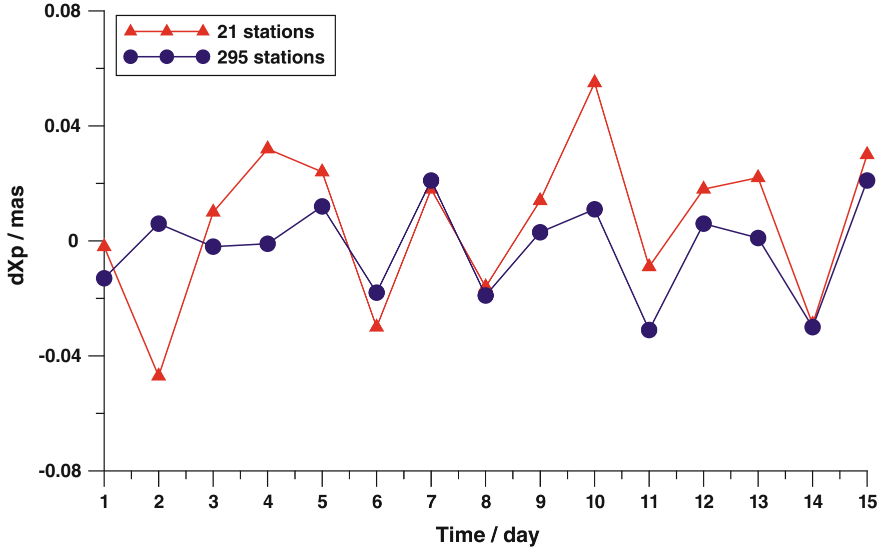

Fig. 24.2 The difference of PM between experimental results and IERS values on X component

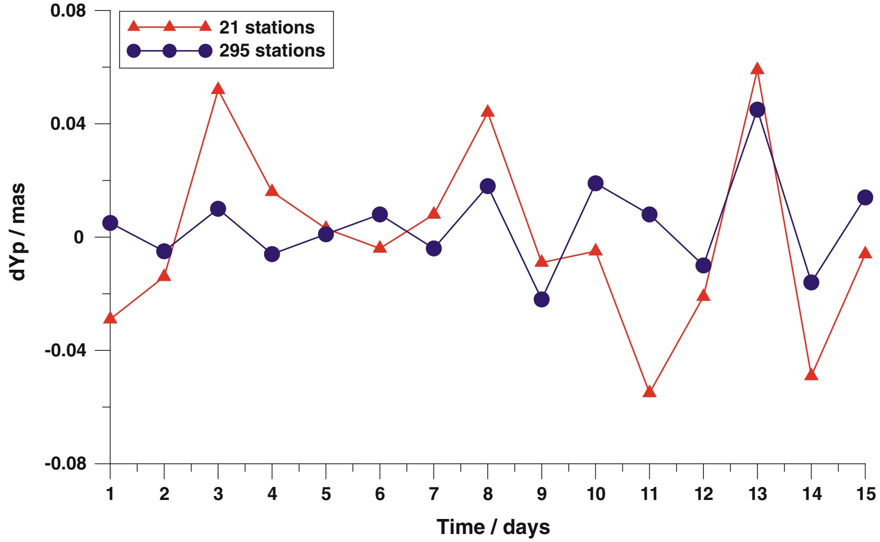

Fig. 24.3 The difference of PM between experimental results and IERS values on Y component

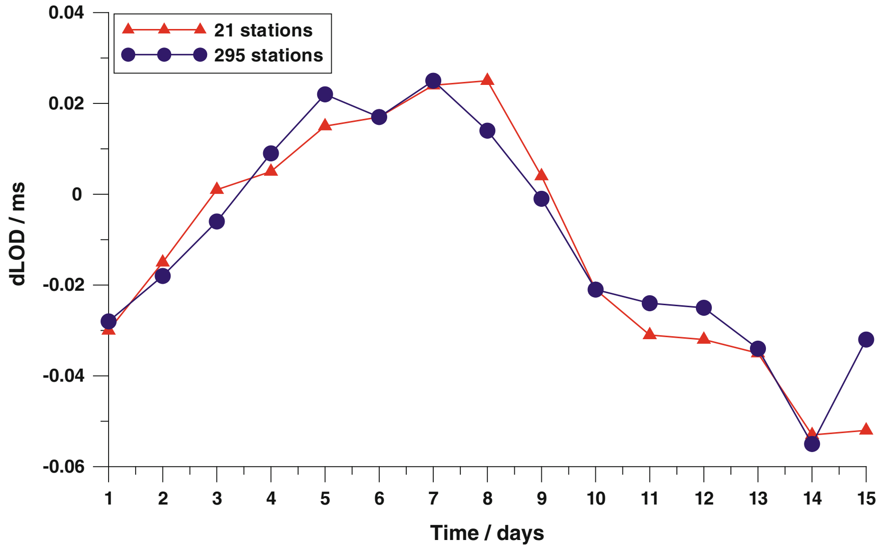

Fig. 24.4 The difference of LOD between experimental results and IERS values

Table 24.1 The precision statistics of two experimental results

	21 stations			295 stations		
	Xp/mas	Yp/mas	LOD/ms	Xp/mas	Yp/mas	LOD/ms
Max.	0.055	0.059	0.053	0.031	0.045	0.055
Min.	0.002	0.003	0.001	0.001	0.001	0.001
Std.	0.028	0.033	0.027	0.017	0.016	0.024

lead the geometry of satellites to change significantly. On the contrary, the location of stations is always invariable. Therefore, it is the decisive factor of ERP estimation precision using GNSS data that the stations distribution.

Because the stations have a rational distribution in two experiments, the precise ERP can be obtained whether 295 or 21 stations data are used. Comparing with the results of IERS, the estimation precisions of pole motion and length of day parameters are better than 0.04 mas and 0.03 ms, respectively

24.5 Conclusions

Based on above theoretical derivation and real data processing, some conclusions can be drawn.

1. There are two major factors which effect ERP estimation precision using GNSS data. One is the geometric distribution of the satellites relative to the station; another is the geometric distribution of the stations relative to the geo-center.

2. Because the long time observation will lead the geometry of satellites to change significantly, it has very light effect on ERP estimation that the geometric distribution of satellites. The location of stations is always invariable. Therefore, it has important effect on ERP estimation that the geometric distribution of stations.
3. The ERP will not be solved if using one GNSS station. The solution precision of ERP will be the best when the vectors of two stations to geo-center are orthogonal. And the solution precision is improving with the value of $\det(A^T A)$ is increasing, if using many stations data to estimate ERP.
4. If the distribution of stations is well, the high precision ERP can be obtain just using a small number of stations data. Therefore, for obtaining precise ERP using Compass satellite data, the key problem is stations geometric distribution rather than the number of stations.

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